Objective Number Theory

The possibility of encoding information about combinatorial objects into the geometry of complex numbers is often described as one of the mysteries of analysis. A demystifying tool was provided by S. H. Schanuel's Como paper (in SLNM 1488) "Negative sets have Euler characteristic and dimension" in the form of the functorial construction called the Burnside rig of a distributive category. Schanuel has recently showed that in a finite separable extension of the category of finite sets, the polynomial equations satisfied by the objects have in the complex numbers only integral roots, but that there are many combinatorial categories to which the result applies, for example directed graphs. But there are also "infinite" examples where abstract equations can be seen as a reflection of objective isomorphisms: for example, the Euler product formula for the zeta function has behind it an isomorphism involving cartesian products of free monoids. My calculation showing that the "space" of binary trees has as Euler characteristic the complex number which is the primitive sixth root of unity, has recently been completed by A. Blass: seven-tuples of trees can be precisely coded as single trees by a very simple bijection, but if "7" is replaced by 5 or any number not congruent to 1 mod 6, such bijections can only be found in a category qualitatively more complicated.